ENTROPY CORRELATION OF CONDENSATION SHOCKS IN HYPERSONIC NOZZLES

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Using an approximate solution [1] a two-parameter correlation is obtained between the maximal supercooling(the Wilson point) and the distribution of gasdynamic parameters in the zone of spontaneous condensation for a hypersonic flow. One parameter is the gas entropy in the injection cup, and the other is the product of the power-law stagnation temperature and the ratio of the characteristic dimension of critical section to the tangent of the nozzle half-angle.

1. Correlation of Maximal Supercooling

In [2] an approximation method was described for evaluating condensation shocks in nozzles, entailing, essentially, the determination of the maximal flow supercooling by employing the procedure of [1]. Maximal supercooling is attained near the Wilson point, its position in the nozzle being determined by

$$dy/dx = dy_e/dx, \tag{1.1}$$

where y and y_e are the nonequilibrium and equilibrium degrees of condensation; x is the distance on the nozzle axis. Using the relations given in [1] for the equilibrium and nonequilibrium condensation rates found in [1] the relation (1.1) can be written as follows:

$$a\frac{\Phi\dot{r}^3}{u^4}\left(1-\frac{\varkappa}{\varkappa-1}\frac{RT}{L}\right)^{-1}\left(\frac{dk}{dx}\right)^{-3} = -\frac{1}{p}\frac{d\ln p}{dx}, \tag{1.2}$$

$$\Phi = \int_0^k (k-t)^2 \exp\left(-t^{-2}\right) dt, \quad k = bT^{3/2}\ln\frac{p}{p_s(T)},$$

where L is the heat of steam generation; R is the gas constant; $\varkappa$ is the adiabatic index; a and b are constants; T is the static temperature at the Wilson point; p is the pressure; $p_s(T)$ is the saturation pressure at the temperature T; and $\dot{r}$ is the growth rate of a droplet [1].

The nozzle geometry and the stagnation parameters appear only implicitly in Eq. (1.2); if the shape of the nozzle is specified exactly, then by using (1.2) a number of new results can be obtained. A class of nozzles given by the formula

$$A = A^0/A^* = (1 + (x/r_*)\,\mathrm{tg}\,\gamma)^i, \quad i = 1, 2. \tag{1.3}$$

is of interest in practice, where A^0 is the cross-sectional area; r_* is the characteristic size of the critical section; and γ is the angle between a generator and the nozzle axis.

The relation (1.1) for the Wilson points is similar to the familiar Bray–Finney criterion [3-5] for the freezing of physicochemical relaxation of a high enthalpy flow. By employing the approach described in [3, 4], the relation (1.2) is transformed to fit the class of nozzles (1.3).

It is known that a rapid parameter change due to condensation begins a little below the Wilson point. This enables us to employ the isentropic condition for flows of a supercooled gas and to express the derivatives of k and p in (1.2) in terms of T and the flow entropy S_0:

$$\frac{dk}{dx} = \frac{dk}{dT}\frac{dT}{dA}\frac{dA}{dx}; \quad \frac{dp}{dx} = \frac{dp}{dT}\frac{dT}{dA}\frac{dA}{dx}, \tag{1.4}$$

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$$\frac{dk}{dT} = bT^{1/2}\left(\frac{3}{2}\ln\frac{p}{p_s} + \frac{\varkappa}{\varkappa - 1} - \frac{T}{p_s}\frac{dp_s}{dT}\right),$$
$$\frac{dp}{dT} = \frac{\varkappa}{\varkappa - 1}\frac{p}{T}, \quad \frac{dA}{dT} = A\frac{(\varkappa + 1)T - 2T_0}{2(\varkappa - 1)T(T_0 - T)},$$
$$dA/dx = i(\mathrm{tg}\,\gamma/r_*)A^{1-1/i},$$
$$A = c(\varkappa)[T_0/(T_0 - T)]^{1/2}(T_0/T)^{1/(\varkappa-1)},$$
$$\ln p = \varkappa/(\varkappa - 1)\ln T - (S_0 - S')/R,$$

where T_0 is the stagnation temperature; S' is an entropy constant. Substituting the relations (1.4) into Eq. (1.2) one obtains an expression of the form

$$c_i\frac{F_i^0(\varkappa, T, S_0)}{2T_0 - (\varkappa + 1)T}(T_0 - T)^{(3i+1)/2i}T_0^{(\varkappa+1)/2i(\varkappa-1)} = r_*/\mathrm{tg}\,\gamma, \tag{1.5}$$

where c_i is a constant depending on the physical properties of the gas; $F_i^0(\varkappa, T, S_0)$ is a function of entropy and of static temperature at the Wilson point.

If our considerations are confined to that range of stagnation parameters for which the saturation state is reached in the hypersonic part of the nozzle, then ignoring the ratio T/T_0 in (1.5), as compared with $2/(\varkappa+1)$, one obtains the final result

$$c_iF_i(\varkappa, T, S_0) = \varphi(r_*, \mathrm{tg}\,\gamma, T_0), \tag{1.6}$$

where

$$\varphi(r_*, \mathrm{tg}\,\gamma, T_0) = (r_*/\mathrm{tg}\,\gamma)T_0^{1/i(\varkappa-1)-1/2}. \tag{1.7}$$

Equation (1.7) determines the static temperature at the Wilson point or the maximal supercooling as a function of the entropy S_0 and the parameter φ. Then in flows with equal values of S_0 the same values of maximal supercooling are reached if the nozzle geometry and the stagnation temperature vary in accordance with the condition φ = const.

The relation (1.6), as well as the original equation (1.2), holds within the framework of the classical condensation theory for any simple gas. For a particular case of entropy correlation (1.6), for flows in conical nozzles ($i = 2$), an experimental verification can be found in [6].

2. Correlation of Distribution of Gasdynamic Functions

The system of gasdynamic equations usually applied to evaluate condensation shocks is given in a one-dimensional approximation by [7]

$$\rho uA + yQ = Q; \quad [(1 - y)/\rho]dp/dx = -u\,du/dx; \tag{2.1}$$
$$u^2/2 + c_pT - Ly = H_0, \quad p = \rho RT,$$

where ρ is the density; Q is the flow rate; H_0 is the stagnation enthalpy; and c_p is the heat capacity.

If the geometry of a nozzle is known, then the system (2.1) can be reduced to the equation

$$\frac{du^2}{2dx} = \frac{c_pT - L(1 - y)\;dy/dx + c_pT(1 - y)(1/A)\,dA/dx}{\varkappa/(\varkappa - 1) - [(1 - y)/u^2](c_pT + u^2)}. \tag{2.2}$$

For the initial condensation stage in the hypersonic flow zone ($y \ll 1$, $c_pT \ll u^2$) Eq. (22) can be reduced to

$$\frac{du^2}{2dx} = \left(\frac{du^2}{2dx}\right)_y + \left(\frac{du^2}{2dx}\right)_a = \frac{\varkappa - 1}{\varkappa}(c_pT - L)\frac{dy}{dx} + \frac{\varkappa - 1}{\varkappa}c_pT\frac{1}{A}\frac{dA}{dx}, \tag{2.3}$$

where the subscripts y and a denote those components of the derivative (2.3) which are due to the phase or the geometric effects, respectively. Substituting (2.3) into the equations of energy conservation, of momentum conservation, and of state for the system (2.1), one is also able to determine the phase components in the derivatives of temperature, pressure, and density. Thus,

$$(du^2/2dx)_y = \psi_1(T)dy/dx; \quad (dT/dx)_y = \psi_2(T)dy/dx; \tag{2.4}$$
$$(dp/dx)_y = \psi_3(T, S_0)dy/dx; \quad (d\rho/dx)_y = \psi_4(T, S_0)dy/dx.$$

According to [2] the rate of phase transition at the initial condensation stage is given by the relation

$$dy/d\tau = \alpha(p/RT)r^3\Phi(dk/d\tau)^{-3},$$

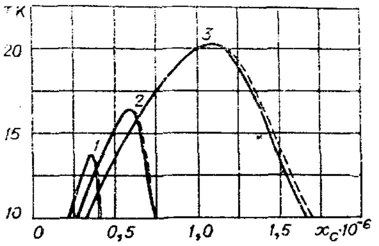

Fig. 1

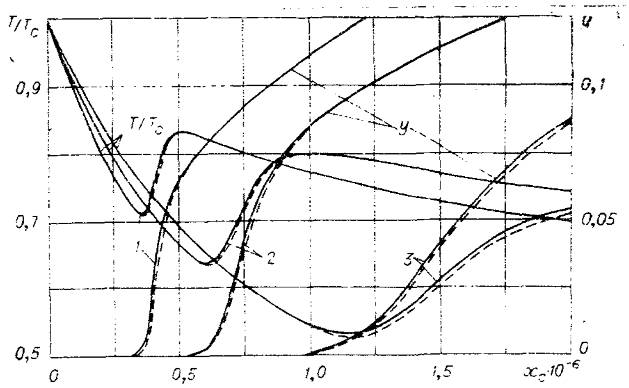

Fig. 2

where α is a constant; Φ is the function of (1.2).

By applying the approach used in Sec. 1 this equation can be rewritten as

$$dy/dx = \alpha(p/RT)(r^3\Phi/u^4)(dk/dT)^{-3}(dA/dT)^3(dA/dx)^3. \quad (2.5)$$

In the hypersonic approximation for the class of nozzles (1.3), Eq. (2.5) together with (1.4) can be reduced to the equation

$$dy/dx = \alpha_i D_i(\varkappa, T, S_0)[(r_* \operatorname{tg} \gamma)T_0^{1/(\varkappa-1)-2/3}. \quad (2.6)$$

By combining the relations (2.4) and (2.6) one comes to the conclusion that at the initial portion of the condensation shock (close to the Wilson point) the phase components of any gasdynamic functions only depend on entropy and on the maximal overcooling provided that the product of x and the parameter

$$\zeta = [(r_*/\operatorname{tg} \gamma)T_0^{1/(\varkappa-1)2/3}]^3.$$

is used as its argument. In accordance with the classic theory, the prehistory of the flow expansion prior to the saturation state has no effect on spontaneous condensation. It is, therefore, obvious that for the variable x the distance to the saturation point x_c^0 must be employed.

These conclusions have been verified by the evaluation of the condensation shocks for conical nozzles. The evaluation was based on the equations in [7]. The growth rate of the drop was determined by using the equation proposed in [2]. Nitrogen was used as generator gas, and the functional dependence of the surface tension coefficient and liquid phase density on temperature was also taken into account [8].

The results for three different values of the entropy [S_0 = 5.3; 5.5; 5.7 J/g·deg (the curves 1-3, respectively)] and for two different values of stagnation temperature [T_0 = 300°K (continuous lines) and 500°K (dashed lines)] are shown in Figs. 1 and 2. The value of the parameter φ was the same in all variants ($\log \varphi$= 2.12; r_*, cm).

In Fig. 1 the supercooling of the flow ΔT is shown and in Fig. 2, the condensation degrees and the ratios T/T_e as functions of the argument $x_c = x_c^0\zeta$; x_c^0, cm (T_c is the temperature at the saturation point).

The results of calculations show a high degree of correlation for the maximal supercooling flow for the parameters S_0 and φ. Moreover, the correlation for the distribution of gasdynamic functions is valid for the initial stage, as well as for the entire spontaneous condensation zone; it also possesses a high degree of accuracy within a wide range of stagnation parameters.

Using the obtained correlation one can construct a simple but sufficiently accurate engineering procedure for calculating condensation shocks.

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